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(10) Louis L. Scharf

Odile Macchi

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PHASE AND SYMBOL SEQUENCE DECODING ON RANDOM WALK PHASE CHANNELS

by

Louis L. Scharf[1]

Odile Macchi[2]

Abstract

Consider the problem of estimating phase and decoding data symbols from baseband data. Assume the phase sequence is a random walk on the circle and the symbols are drawn independently from an equiprobable alphabet for transmission over a perfectly equalized channel. A dynamic programming algorithm (Viterbi algorithm) is derived for decoding a maximum a posteriori (MAP) phase-symbol sequence on a finite dimensional phase-symbol trellis. Simulation results for binary and 8-ary phase modulated (PM) symbol sets are presented.

I. Introduction

Current techniques for decoding data symbols transmitted over random phase channels include the decision-directed phase-lock loop (DDPLL) of [1] and the decision-directed stochastic approximation procedure of [2]. The potential of maximum a posteriori (MAP) sequence estimation for improving on the DDPLL has been recognized in [3], where the author derives a path metric and indicates its rôle in a forward dynamic programming algorithm for obtaining MAP phase-symbol sequences. However, because of the way random phase is modelled in [3], two simplifying assumptions must be made in order to implement a tractable, finite dimensional algorithm.

In this paper we observe that only phase-modulo 2π is of interest in data communication applications. This motivates us to wrap the phase around the circle, so to speak, and obtain a folded normal model for transition probabilities in random walk on the circle. It is then straight-forward to pose a MAP phase-symbol sequence estimation problem as in [5] and [6]. The basic idea is to discretize the phase space $[-\pi,\pi)$ to a finite dimensional grid and to use a dynamic programming algorithm (Viterbi algorithm) to keep track of survivor phase-symbol sequences that can ultimately approximate the MAP sequence.

II. Model for Symbol Transmission over a Random Walk Phase Channel

As our model for symbol transmission over a perfectly equalized channel we assume

$$z_k = i_k e^{j\phi_k} + n_k, \quad k = 1,2,\ldots \tag{1}$$

Here $\{z_k\}$ is a sequence of baseband measurements, $\{i_k\}$ is an independent sequence of M-ary complex symbols drawn from an equiprobable alphabet, $\{\phi_k\}$ is a random walk phase sequence, and $\{n_k\}$ is an additive noise sequence. The sequences $\{i_k\}$, $\{\phi_k\}$, and $\{n_k\}$ are independent of each other.

The phase is modelled as a random walk on the circle taking values in $[-\pi,\pi)$. This sequence is Markov with transition probabilities characterized by the following folded-normal law [4], [5]:

$$f(\phi_k/\phi_{k-1}) = g_1(\phi_k-\phi_{k-1})$$

$$g_M(x) = M^{-1} \sum_{m=1}^{M} \sum_{\ell=-\infty}^{\infty} \frac{1}{(2\pi\sigma_w^2)^{1/2}} \exp\{-\frac{1}{2\sigma_w^2}[x-\ell 2\pi-(m-1)\cdot 2\pi/M]^2\} \tag{2}$$

We may think of $\{\phi_k\}$ as a modulo-2π version of usual random walk. The function $g_M(x)$ is useful for our discussion of PM in Section IV.

The noise sequence is assumed to be a sequence of complex normal random variables:

$$n_k = u_k + jv_k \qquad u_k \perp\!\!\!\perp v_\ell \;\; \forall(k,\ell)$$

$$u_k : N(0,\sigma_n^2) \qquad v_k : N(0,\sigma_n^2) \tag{3}$$

$$u_k \perp\!\!\!\perp u_\ell, \;\; k \neq \ell \qquad v_k \perp\!\!\!\perp v_\ell, \;\; k \neq \ell$$

Here $u_k : N(0,\sigma_n^2)$ indicates u_k is normal with mean zero and variance σ_n^2. It follows that the joint density of the observations $\{z_k\}_1^K$, phases $\{\phi_k\}_1^K$, and symbols $\{i_k\}_1^K$ may be written

$$f(\{z_k\}_1^K, \{\phi_k\}_1^K, \{i_k\}_1^K) = M^{-K} \prod_{k=1}^{K} N_{z_k}(i_k e^{j\phi_k}, \sigma_n^2) g_1(\phi_k-\phi_{k-1}) f(\phi_1) \tag{4}$$

where $f(\phi_1)$ is the marginal density of ϕ_1 and where $N_{z_k}(x,\sigma_n^2)$ denotes the complex normal density $(2\pi\sigma_n^2)^{-1} \exp\{-|z_k-x|^2/2\sigma_n^2\}$. The joint density of (4) is actually a mixed density-probability mass function defined on $R^K \times [-\pi,\pi)^K \times I^K$ where R is the real line and I is the M-dimensional discrete-set of source symbols.

III. A Viterbi Algorithm for MAP Phase and Symbol Sequence Decoding

Consider the following maximization problem with respect to the phase sequence $\{\phi_k\}_1^K$ and the symbol

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sequence $\{i_k\}_1^K$:

$$\max_{\{\phi_k\}_1^K, \{i_k\}_1^K} f(\{z_k\}_1^K, \{\phi_k\}_1^K, \{i_k\}_1^K) \quad . \quad (5)$$

Maximization of this joint density function is equivalent to maximization of the a posteriori density $f(\{\phi_k\}_1^K, \{i_k\}_1^K / \{z_k\}_1^K)$, so we call the maximizing sequences $\{\hat{\phi}_k\}_1^K$, $\{\hat{i}_k\}_1^K$ the MAP phase-symbol sequences. If instead we maximize the $\ell n\ f(., .,.)$ and ignore uninteresting constants we have the problem

$$\max_{\{\phi_k\}_1^K, \{i_k\}_1^K} \Gamma_K \qquad (6)$$

where Γ_K is defined recursively in terms of the path metric p_k:

$$\Gamma_k = \Gamma_{k-1} + p_k$$

$$p_k = -\frac{1}{2\sigma_n^2} |z_k - i_k e^{j\phi_k}|^2 + \ell n\ g_1(\phi_k - \phi_{k-1}) \qquad (7)$$

$$\Gamma_0 = 0, \ p_1 = -\frac{1}{2\sigma_n^2} |z_1 - i_1 e^{j\phi_1}|^2 + \ell n\ f(\phi_1) \quad .$$

In this form the maximization problem may be efficiently solved by using a dynamic programming algorithm (or Viterbi Algorithm) on a finite-dimensional grid of phase-symbol pairs. The reader is referred to [5] and [6] for details.

IV. An Optimum Decoding Algorithm for Phase-Coded Symbols

Suppose $I = \{e^{j2\pi(m-1)/M}\}_{m=1}^M$ corresponding to equally-spaced PM symbols with unit energy. Write

$$z_k = e^{j\psi_k} + n_k \ , \quad k = 1,2,\ldots$$
$$\psi_k = \phi_k + \theta_k \quad , \quad \theta_k \in I \qquad (8)$$

and consider the following MAP sequence estimation problem:

$$\max_{\{\psi_k\}_1^K, \{\theta_k\}_1^K} f(\{z_k\}_1^K, \{\psi_k\}_1^K, \{\theta_k\}_1^K) \qquad (9)$$

Assuming $\{\psi_k\}$ to be defined on $[-\pi,\pi)$ one can show that the joint density in (9) may be written

$$f_K = \prod_{k=1}^K N_{z_k}(e^{j\psi_k}, \sigma_n^2)\ M^{-1}\ g_1(\psi_k - \psi_{k-1} - (\theta_k - \theta_{k-1})). \qquad (10)$$

Call $\{\hat{\psi}_k\}_1^K$ and $\{\hat{\theta}_k\}_1^K$ the MAP sequences that jointly maximize f_K. These sequences enter jointly only in the $g_1(\cdot)$ term on the right hand side of (10). It follows then that f_K is minimized by choosing.

$$\hat{\theta}_k = \hat{\theta}_{k-1} + [\hat{\psi}_k - \hat{\psi}_{k-1}] \qquad (11)$$

where $[\cdot]$ denotes the closest value of $(m-1)\ 2\pi/M$ to $\hat{\psi}_k - \hat{\psi}_{k-1}$ and $\hat{\theta}_k$ is interpreted modulo-2π. It follows f_k may be rewritten as

$$f_K = \prod_{k=1}^K N_{z_k}(e^{j\psi_k}, \sigma_n^2)\ M^{-1}\ g_1(R(\psi_k - \psi_{k-1})) \qquad (12)$$

where $R(x)$ is the difference between x and $[x]$. Thus we may maximize (12) with respect to $\{\psi_k\}_1^K$, and then find $\{\hat{\theta}_k\}_1^K$ from (11). In a sense, we are decoding $\{\psi_k\}^K$, as if it had a "density" given by $M^{-1}\ g_1(R(\cdot))$ and simply using the algorithm of [5] and [6]. When the overlap between $2\pi/M$ translates of $g_1(\psi_k - \psi_{k-1})$ is negligible, then $g_M(\psi_k - \psi_{k-1}) \doteq M^{-1}\ g_1(R(\psi_k - \psi_{k-1}))$ and we may write the problem of maximizing $\ell n\ f_K$ as in (6) and (7) with p_k replaced by p_k':

$$p_k' = -\frac{1}{2\sigma_n^2} |z_k - e^{j\psi_k}|^2 + \ell n\ g_M(\psi_k - \psi_{k-1}) \quad . \quad (13)$$

The function g_M is the convolution of the folded phase density with the impulsive density of the data.

V. Results

The phase space $[-\pi,\pi)$ has been discretized to 48 phase values and a Viterbi algorithm has been programmed to solve (9) along the lines outlined in (11)-(13). Initial phase acquisition has been achieved by sending a 100-symbol preamble followed by 900 data symbols. This procedure has been repeated from 10 to 20 times to obtain the performance results of Figs. 1 and 2. When counting symbol errors, bursts have been ignored. Shown in Fig. 1 are binary symbolling results for $\sigma_w^2 = 0.01\ \text{rad}^2$ ($\sigma_w = 5.7^o$) and SNR $\triangleq 10 \log_{10} 1/2\sigma_n^2$ ranging from 4 to 10dB. The results for binary orthogonal symboling are of no inherent interest in their own right. They are presented simply to validate the simulation. The results for binary orthogonal symboling are interesting because they indicate performance nearly equivalent to a receiver that has perfect phase coherence. The circles of Fig. 2 represent error probabilities for 8-ARY PM symboling when SNR ranges from 16-19dB and $(\sigma_n^2\ \sigma_w^2)^{1/2}$ remains fixed at $4.4 \times 10^{-3}\ \text{rad}^2$. The triangles represent error probabilities for the markedly simpler decision-directed algorithm of [2]. For pure PM and moderate values of the phase variance parameter σ_w^2, there seems to be little to recommend the algorithm of Sections III and IV over the algorithm of [2]. A similar conclusion with regard to the DDPLL was reached in [3]. This conclusion is changed for AM-PM symbol constellation such as 16-QASK, 4A-4ϕ, etc.

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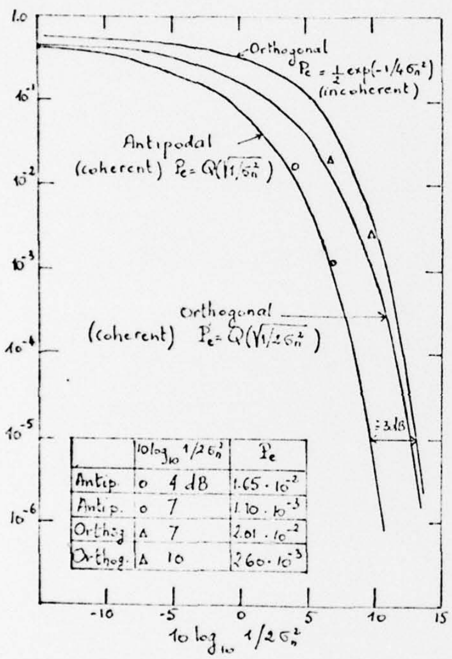

Fig. 1. Symbol Error Probabilities for Binary Symboling.

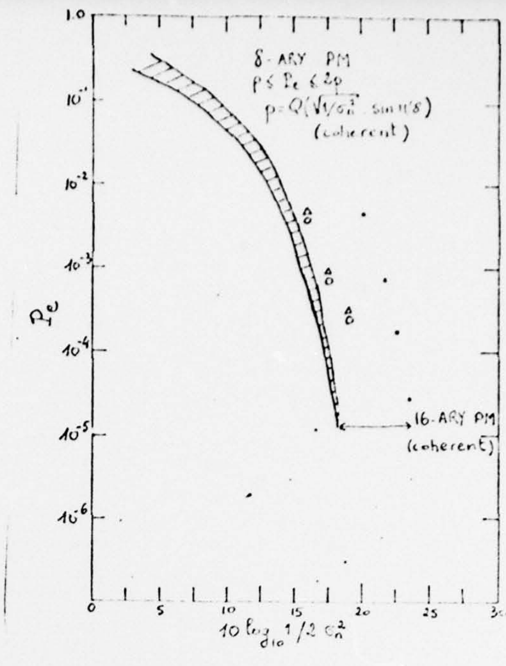

Fig. 2. Symbol Error Probabilities for 8-ary PM.

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[1]Electrical Engineering Department, Colorado State University, Ft. Collins, CO, USA 80523, and Laboratoire des Signaux et Systèmes, Plateau du Moulon, 91190, Gif-sur-Yvette, France.

[2]Laboratoire des Signaux et Systèmes, Plateau du Moulon, 91190, Gif-sur-Yvette, France.